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The Optimal Timing of Cross-training under Demand Uncertainty: A Real Options Approach

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Abstract. Improving labor flexibility through cross-training is one of the effective ways to respond to demand fluctuation. In this paper, the optimal timing of cross-training is investigated considering demand uncertainty, and labor flexibility is measured by redundancy level. Cross-training can be treated as investment in labor flexibility, so a trinomial lattice based on real options valuation is employed to model the evolution of demand. A stochastic dynamic programming model is formulated. With a numerical example, the proposed approach is proved to be effective in finding the optimal training plan with the consideration of all possible demand scenarios. Results of the numerical example also provide a way to estimate the value of labor flexibility.

Keywords: cross-training, labor flexibility, redundancy level, real options

1. Introduction

Cross-training is a way to quickly improve productivity and flexibility with rather low investment, especially for assembly lines, U-shaped lines, and manufacturing cells [1,2]. Multi-skilled workers can effectively mitigate the influence of demand uncertainty, but overtraining has a negative effect on production efficiency and product quality as well. Therefore, it is important to determine how many workers should be cross-trained and how many skills each worker should master.

A lot of research has investigated how to improve flexibility through reasonable cross-training and proposed some cross-training strategies, such as cherry picking, D-skill chaining, and closed chain [3-7]. These strategies are useful for analyzing the effect of cross-training on flexibility. Liu et al. [8] investigated the cross-training and worker assignment problem when a conveyor assembly line is entirely reconfigured into several serus, and they proposed a heuristic algorithm to solve the two problems simultaneously. Azizi and Liang [9] proposed a two-phase heuristic algorithm for solving task assignment and training problem of multi-skilled workers in cellular manufacturing considering the impact of task rotation on skill level. Nembhard et al. [10] investigated cross-training problem in a sequential production line with two workstations considering task heterogeneity, worker heterogeneity, labor dynamics and product dynamics. Yang and Gao [11] investigated the adjacent workforce cross-training policy in a flexible mixed-model assembly line. A real option valuation technique is employed by Qin and Nembhard [12] to optimize the design of workforce agility for maximum expected return in a stochastically diffused environment.

The flexibility of cross-trained workers can be defined by means of three concepts: multifunctionality, redundancy and functional flexibility [13]. The aforementioned research mainly focuses on multifunctionality, that is, the number of different skills a worker should be trained. This study investigates cross-training from the viewpoint of redundancy. A real options approach is employed to determine the number of workers should be trained for a specific skill and the optimal timing of training each worker.

2. Problem Description

We consider a two-phase cross-training problem with uncertain demand. The labor flexibility is measured by redundancy level, so we want to find out how many workers should be trained for a skill. Generally, the productivity of a product is determined by the number of workers who master the key skill. In

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this problem, workers must finish two separate training phases, basic training and specific training, in order to master the key skill. The two phases must be implemented sequentially, because basic training teaches some fundamental techniques. Therefore, the productivity of a product approximately depends on the number of workers who have finished the two phases.

In order to make quick response to demand fluctuation, more workers can finish their basic training firstly. A few of them start their specific training immediately after the basic training to satisfy current demand. Whether the remainder will start their specific training mainly depends on future demand. Apparently, costs related to basic training increase. In fact, extra basic training creates an opportunity to quickly expand productivity once the demand increases. Therefore, costs paid for extra basic training can be treated as an investment on flexibility, and the manufacturer gains a real option for expansion.

Notations used for the formulation of the problem are listed as follows:

t index of time periods;

T the number of time periods;

M the number of workers;

n_e the number of workers who have already finished two training phases;

V available capacity of a worker;

t_u unit processing time of the key skill;

P price of the product;

C_f unit cost of the product;

C_s hourly pay for workers;

C_b costs of basic training;

C_e costs of specific training;

C_p penalty cost;

f inflation rate;

r discount rate.

Assuming M workers are available, the capacity of each worker is V, and the product's lifespan is T. After T time periods, it will withdraw from the market and its key skill can not be used to process other products. There are n_e workers who have finished two training phases. If capacity of the n_e workers can not satisfy the demand, there will be a penalty cost. Decision variables are the optimal number of workers participate basis training when t=0 (N), the number of workers finished two training phases when t=0 (n_0, $n_0 \leq N$), and the optimal number of workers finished two training phases in each period (n_t, t=1, …, T-1). n_t can be treated as the expansion decision made at the beginning of each period.

Assuming demand over [t, t+1] is d_t, t=0,…,T-1, a cash flow stream for the net profit of the product (x_o, x_1,…, x_{T-1}) can be identified based on N and n_t. At t=0, N and n_0 should be decided, the related training costs are

$$x_0 = -C_b N - C_e n_0. \tag{1}$$

Assuming the specific training would take one time period, so the productivity over [t, t+1] is determined by n_{t-1}. At any t>0, the total net profit accumulated over [t, t+1] is

$$\begin{aligned} x_t = [&P \cdot \min(d_t, \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor) - C_s \cdot \min(d_t t_u/60, (n_{t-1}+n_e)V) \\ &- C_f \cdot \min(d_t, \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor) - C_e(n_t - n_{t-1}) - C_p \cdot \max(0, d_t \\ &- \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor)](1+f)^t \end{aligned} \tag{2}$$

where the five parts represent revenue, payment, product costs, specific training cost and penalty cost respectively.

The expansion decision n_t is subject to

$$n_0 \leq n_1 \leq \cdots \leq n_{T-1} \leq N \leq M. \tag{3}$$

Based on (x_o, x_1,…, x_{T-1}), NPV of the net profits can be determined by

$$NPV = \sum_{t=0}^{T} \frac{x_t}{(1+r)^t}. \quad (4)$$

3. A Real Options Approach

The demand of product is dt over [t, t+1], its logarithmic form is $D_t = ln d_t$. D_t is assumed to follow the following stochastic process:

$$dD_t = \mu(D_t, t)dt + \sigma dB_t \quad (5)$$

where B_t is Wiener process, μ is drift function, σ is the volatility.

The trinomial lattice model is employed to approximate the uncertainty characterized by (5) [14]. In the trinomial lattice, each demand node branches out into three demand nodes. The lifespan of product is divided into several time periods, and the demand may increase or decrease with a constant ΔD during each time period. At time period t, the logarithmic demand is D_t, it branches out into three possible values, $D_t + (k+1)\Delta D$, $D_t + k\Delta D$, and $D_t + (k-1)\Delta D$, with probabilities p^u, p^m, p^d. The constant $\Delta D = \theta\sigma$, where θ is a constant. The branching factor k is an integer, and the value of k is chosen such that $k\Delta D$ can best approximate the expected drift $\mu(D,t)$ [14].

$$k \equiv \left\lfloor \frac{\mu(D,t)}{\Delta D} + \frac{1}{2} \right\rfloor \quad (6)$$

The branching probabilities p^u, p^m, p^d are determined using the following equations [15].

$$p^u(k+1)\Delta D + p^m k\Delta D + p^d(k-1)\Delta D = \mu(D,t) \quad (7)$$

$$p^u(k+1)^2\Delta D^2 + p^m k^2\Delta D^2 + p^d(k-1)^2\Delta D^2 = \sigma^2 + \mu(D,t)^2 \quad (8)$$

$$p^u + p^m + p^d = 1 \quad (9)$$

The value of p^u, p^m, p^d is between 0 and 1, so θ must be chosen between $2/\sqrt{3}$ and 2 [16].

At time period t, D_t branches into D_{t+1}^u, D_{t+1}^m, and D_{t+1}^d, the corresponding cash flows are $x_t(N, n_{t-1}, d_t)$, $x_{t+1}(N, n_t, d_{t+1}^u)$, $x_{t+1}(N, n_t, d_{t+1}^m)$, $x_{t+1}(N, n_t, d_{t+1}^d)$. The recursive relation among the three cash flows is

$$\begin{aligned} x_t(N, n_{t-1}, d_t) = {} & [P \cdot \min(d_t, \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor) - C_s \cdot \min(d_t t_u/60, (n_{t-1}+n_e)V) - C_f \\ & \cdot \min(d_t, \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor) - C_p \cdot \max(0, d_t - \lfloor 60(n_{t-1}+n_e)V/t_u \rfloor)](1+f)^t \\ & + \max_{n_{t-1} \le n_t \le N} \{(1+r)^{-1} [p_t^u x_{t+1}(N, n_t, d_{t+1}^u) + p_t^m x_{t+1}(N, n_t, d_{t+1}^m) \\ & + p_t^d x_{t+1}(N, n_t, d_{t+1}^d)] - C_e(n_t - n_{t-1})(1+f)^t\} \end{aligned} \quad (10)$$

with boundary conditions

$$x_T(N, n_{T-1}, d_T) = 0 \qquad \forall n_{T-1}, d_T. \quad (11)$$

Running the recursive functions from $t = T$ to $t = 0$. At $t = 0$, we get x_0 using the following equation.

$$x_0 = \max_{N, n_0}\{-C_b N - C_e n_0 + (1+r)^{-1}[p_0^u x_1(N, n_0, d_1^u) + p_0^m x_1(N, n_0, d_1^m) + p_0^d x_1(N, n_0, d_1^d)]\} \quad (12)$$

The optimal redundancy level is obtained by finding the N and n_0 that result in maximal x_0.

4. A Numerical Example

Tab. 1 shows the historical demand of product A during the past twelve months, and it will withdraw from the market after eighteen months. Only one worker has finished the two training phases, his/her available capacity is 40 hours/month. It takes 8 minutes to process one product at the task corresponding to the key skill. It is clear that one worker can not satisfy the demand. Therefore, the proposed approach is employed to determine how many workers should start basic training, how many workers should start specific training and when they will start specific training.

Assuming D_h' follow the stochastic process described in (5), and it is easy to get the logarithmic demand D_h' based on d_h', $h = 1, \cdots, H$ given in Tab. 1. $\mu(\cdot)$ is assumed to be a constant μ because we don't have enough data. Using (13) and (14), the estimate of μ and σ^2 are 0.0410 and 0.0043 respectively, and $\Delta D = \theta\sigma = \sqrt{3} \times 0.06525 = 0.1130$. According to (6)-(9), k=0, $p^u = 0.4156$, p^m=0.5316, $p^d =$

0.0528. Fig. 1 depicts the trinomial lattice that shows evolution of the demand in the future, and there are totally 3^{18} possible demand scenarios.

$$\mu = \frac{1}{H}\sum_{h=0}^{H-1}(D'_{h+1} - D'_h) \quad (13)$$

$$\sigma^2 = \frac{1}{(H-1)}\sum_{h=0}^{H-1}(D'_{h+1} - D'_h - \mu)^2 \quad (14)$$

Table 1: The Historical Demand of Product A

Month	Demand	Month	Demand
1	309	7	415
2	346	8	408
3	322	9	432
4	358	10	463
5	384	11	456
6	370	12	485

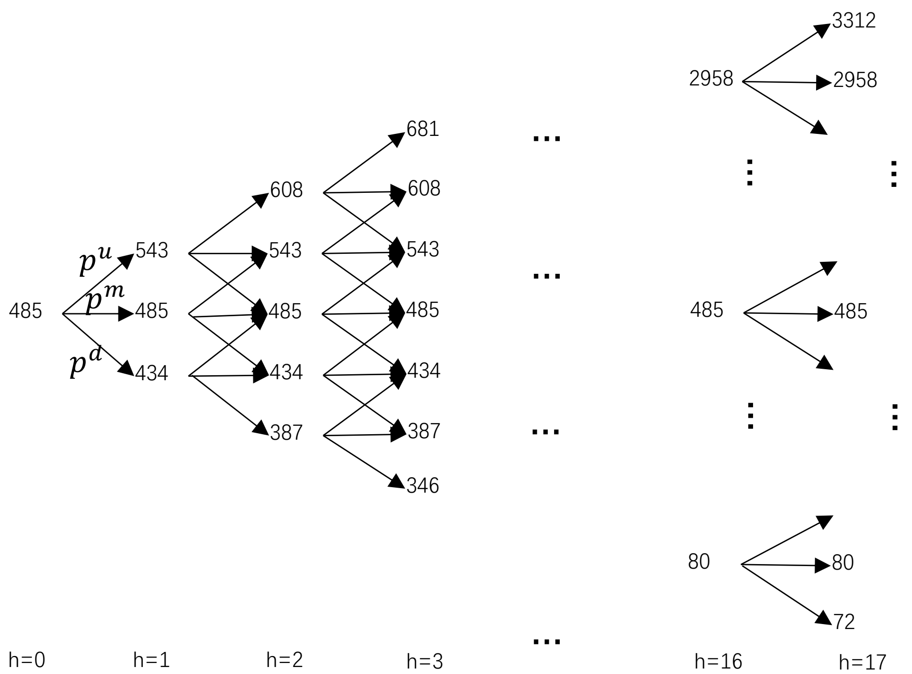


Fig. 1: The trinomial lattice.

Tab. 2 shows value of parameters, and Table 3 shows the expected NPV under different combinations of N and n_0. Taking all possible demand scenarios into consideration, the optimal result is N=6, n_0=1. At the beginning (t=0), 6 workers finish basis training, and one of them finish specific training. The other five workers provide flexibility to quick response to demand increase in the future.

Table 2: Value of Parameters

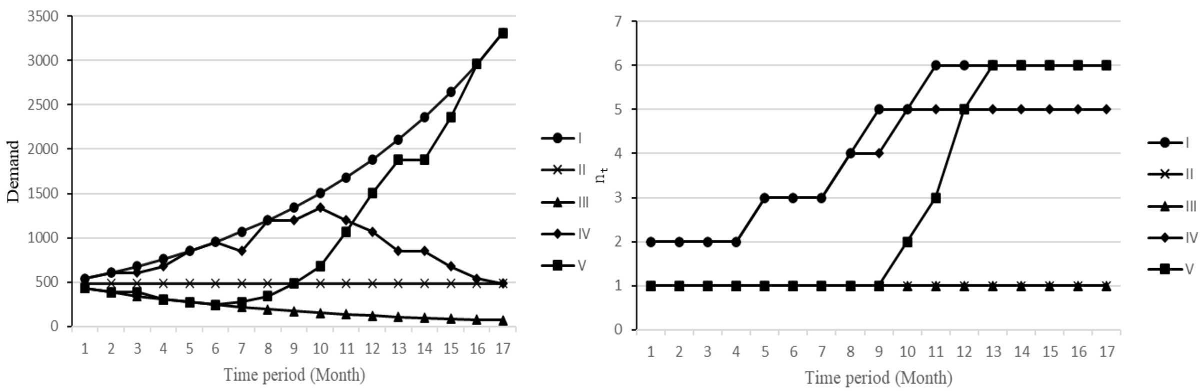
Symbol	Value	Symbol	Value
T	18 months	P	800 yuan/unit
M	10 workers	C_s	50 yuan/hour
n_e	1 worker	C_f	300 yuan/unit
V	40 hours/month	C_b	1000
t_u	8 minutes/unit	C_e	3000
f	5%	C_p	800 yuan/unit
r	8%		

Fig. 2 shows details of specific training plan under five demand scenarios. In scenario I, demand increases continuously, the remained five workers all finished the specific training, and they are trained in the 1st, 5th, 8th, 9th, and 11th month respectively. In scenario II and III, none of the remained five workers starts the specific training. In scenario IV, demand increases firstly, and then decreases. Therefore, three workers finished the specific training in the 1st, 5th, 8th, and 10th month, and no more worker starts the specific training after that. In scenario V, demand decreases firstly, and then increases dramatically. Therefore, no one starts specific training until the 10th month. In the 12th month, two workers start specific training to response to the large increase in demand.

We can also estimate the valuation of flexibility using results in Tab. 3. If N=n_0, no extra worker can start specific training in the future, it means there is no flexibility to respond demand increase. If N>n_0, workers who only finished basic training provide flexibility to respond demand increase quickly. Therefore, the differences between their expected NPV can be treated as the value of labor flexibility. For example, the expected NPV of (N=6, n_0=1) is 4.6641 billion yuan, the expected NPV of (N=1, n_0=1) is 2.2952 billion yuan, so value of the five more workers kept for future use is 2.3689 billion yuan, even higher than the expected NPV of (N=1, n_0=1).

Table 3: The Expected NPV (￥10,000) under Each Combination

N	***n*₀**									
	1	2	3	4	5	6	7	8	9	10
1	229.52									
2	411.33	411.31								
3	457.15	457.13	456.99							
4	465.23	465.21	465.08	464.83						
5	466.31	466.30	466.16	465.91	465.63					
6	**466.41**	466.40	466.26	466.01	465.73	465.43				
7	466.33	466.32	466.18	465.93	465.65	465.35	465.05			
8	466.23	466.22	466.08	465.83	465.83	465.25	464.95	464.65		
9	466.13	466.12	465.98	465.73	465.45	465.15	464.85	464.55	464.25	
10	466.03	466.02	465.88	465.63	465.35	465.05	464.75	464.45	464.15	463.85



(a) Demand scenarios (b) Specific training plans

Fig. 2: Specific training plans under different demand scenarios.

5. Conclusion

In this paper, we consider a two-phase cross-training problem with uncertain demand. The problem is solved from the viewpoint of redundancy to determine how many workers should be trained for certain skill and the optimal timing of training. A stochastic dynamic programming model is formulated. To deal with demand uncertainty, a real options approach is employed. Then, a numerical example is tested by the proposed approach. The results show that the approach not only provides the optimal training plan with the consideration of all possible demand scenarios, but also provides the training plan under each demand scenario. Based on the results of different combinations of N and n0, the value of labor flexibility can also be estimated.

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7. References

[1] R. Qin, D.A. Nembhard, W.L. Barnes II. Workforce flexibility in operations management. *Surv. Oper. Res. Mana. Sci.*, 2015, **20**: 19-33.

[2] W.J. Hopp, and M.P. Van Oyen. Agile workforce evaluation: a framework for cross-training and coordination. *IIE Trans.* 2004, **36**(10): 919-940.

[3] Z. Aksin, M. Armony, V. Mehrotra. The modern call center: A multi-disciplinary perspective on operations management research. *Prod. Oper. Mana.* 2007, **16**(6): 665-688.

[4] T.R. Robbins, D.J. Medeiros, T.P. Harrison. Cross training in call centers with uncertain arrivals and global service level agreements. *Int. J. Oper. Quant. Mana.* 2010, **16**(3): 307-329.

[5] S.M. Iravani, M.P. Van Oyen, K.T. Sims. Structural flexibility: A new perspective on the design of manufacturing and service operations. *Mana. Sci.*, 2005, **51**(2): 151-166.

[6] W.J. Hopp, E. Tekin, M.P. Van Oyen. Benefits of skill chaining in serial production lines with cross-trained workers. *Mana. Sci.* 2004, **50**(1): 83-98.

[7] H. Parvin, M.P. Van Oyen, D.G. Pandelis, D.P. Williams, J. Lee. Fixed task zone chaining: worker coordination and zone design for inexpensive cross-training in serial CONWIP lines. *IIE Trans.* 2012, **44**: 894-914.

[8] C.G. Liu, N. Yang, W. Li, J. Lian, S. Evans, Y. Yin. Training and assignment of multi-skilled workers for implementing seru production systems. *Int. J. Adv. Manuf. Tech.* 2013, **69**(5-8): 937-959.

[9] N. Azizi, and M. Liang. An integrated approach to worker assignment, workforce flexibility acquisition, and task rotation. *J. Oper. Res. Soc.* 2013, **64**: 260-275.

[10] D.A. Nembhard, H.B. Nembhard, R. Qin. A real options model for workforce cross-Training. *Engin. Econ.* 2005, **50**(2): 95-116.

[11] C. Yang, and J. Gao. Balancing mixed-model assembly lines using adjacent cross-training in a demand variation environment. *Comput. Oper. Res.* 2016, **65**: 139-148.

[12] R. Qin, and D.A. Nembhard. Workforce agility for stochastically diffused conditions-a real options perspective. *Int. J. Prod. Econ.* 2010, **125**: 324-334.

[13] E. Molleman, and J. Slomp. Functional flexibility and team performance. *Int. J. Prod. Res.* 1999, **37**(8): 1837-1858.

[14] T. Zhao, and C. Tseng. Valuing flexibility in infrastructure expansion. *J. Infr. Syst.* 2003, **9**(3): 89-97.

[15] C. Tseng, and K.Y. Lin. A framework using two-factor price lattices for generation asset valuation. *Oper. Res.* 2007, **55**(2): 234-251.

[16] J.C. Hull, and A. White. One-factor interest-rate models and the valuation of interest-rate derivatives securities. *J. Financ. Quant. Anal.* 1993, **28**(2): 235-254.